Newton and the Pshooter Gang

Explorations in Classical Physics

Revision Dated: 26 January 2009

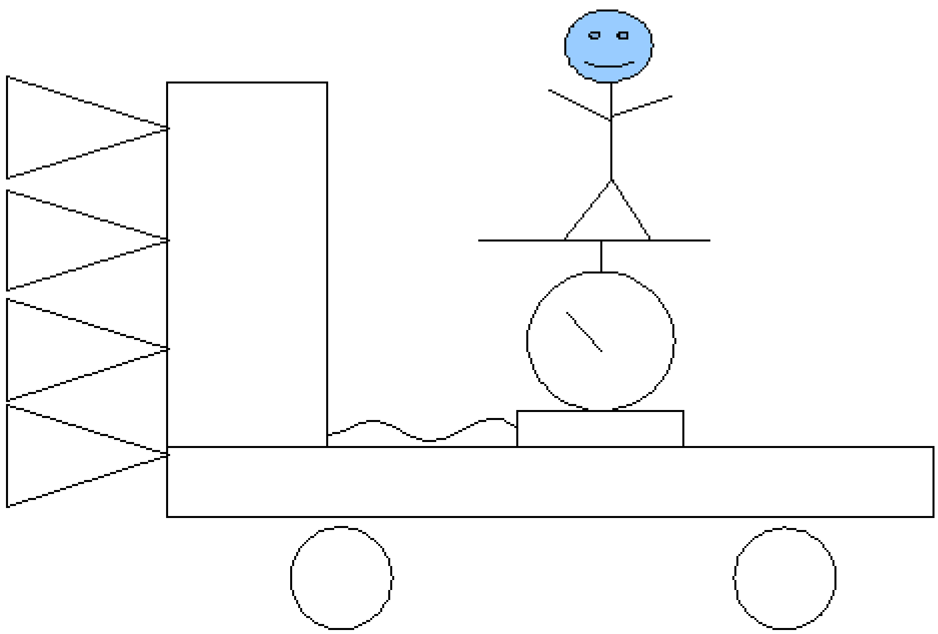


Table of Contents

Introduction....5
Basic Units of Measurement....5
Derived units: Momentum....6
- Suggested Unit Name for Momentum....6
- Newton's First Law, the Law of Inertia....6
- Law of Inertia, Intuitive Example....7
- Law of Conservation of Momentum....7
- Conservation of Momentum Intuitive Examples....8
- Newtons Second Law: Force = Mass * Acceleration....8
- Newton's Third Law: Every Force has a -Force....9
- Newton's Laws: Intuitive Example Air Hockey....10
- Newton's Laws of Motion: Roller Skating Tug Of War....12
- Some Observations from the Example....13
- Example: Firing One Bullet....13
- Cannon on a Cart....14
- The Pshooter Cannon on a Cart....16

Derived Unit: Force Measured Newtons....19
- The pshooter....19
- Quick Review: Proving Newton's Second Law....20

The Pshooter Races....20
- Do you know that girl?....22
- What is Gravity?....22
- The Earth as a Billiard Ball....23
- The Earth as Buckaroo's Skateboard....24
- Rest Stop: Gravity is Pshooters per Kilogram....25

Derived Unit: Work = Force applied over Distance = Joules....26
- Work Lifting Weights....27
- Work Pushing Mass With No Friction....27

Derived Unit: Energy = the ability to do work = Joules....29
- Potential Energy....29
- Kinetic Energy....30
- Law of Conservation of Energy....30
- Comparing Potential and Kinetic Energy: Superball....31

Derived Unit: Power = The Rate at Which Work is Done = Watts....34
- Powered Deck of Cards....34

Power is a Mass Moving at Constant Vertical Velocity....35
But Kilowatt-hour is Energy....36
Review: Impulse,Momentum,Force,Work,Energy,Power....37
Review, Part 2: For Us Earth Bound Types....37
Complicated Collisions: Momentum and Energy....38
Assuming Puck Stops After Collision....39
Two Equations, Two Unknowns....40
Somewhat Non Intuitive Results....42
Velocity of two masses after Elactic Collision....43
Elastic Versus Inelastic Collisions....43
Inertial Frames....44
Rotational Momentum....45
Rotational Kinetic Energy....45
Resources....46
Shots Per Second Calculator....47
English Conversion Units....49
Length....49
Area:....50
volume:....51
Weight (Avoirdupois system)....52
Weight (troy system)....53
Oddball conversions....54
1 foot pound....54
1 horsepower....54
British Thermal Units....54
1 atmosphere....55

1 January 2009: first draft

26 January 2009: fix Leibniz bio

Introduction

This document is intended as a introduction to classical physics for people who've never explored it and as a quick refresher for those who've forgotten it. We'll cover the basic units of measurement in physics (mass, distance, time) and then from there we'll derive a number of other units such as impulse, momentum, and force. Then we'll move onward to define work, energy, and power, among others. And we'll try to show how they all relate to one another and how to use them to solve problems in classical physics.

By “classical physics” we're talking about physics without special relativity. This means when we talk about velocities of something, it's well below the speed of light.

We're talking about problems involving everyday objects. Two billiard balls collide on a billiard table, how fast will each be moving after they collide? If I drop an apple from a certain height, how fast will it be moving when it hits the ground? How much work will it take to lift a deck of cards and put them on a table? I have a 1 horsepower winch lifting some weight on a rope, how fast will it lift it? What is that watermelon doing there? And so on.

You should be able to solve these kinds of problems by the time you finish reading this document.

For the history buffs reading this, classical physics is just another way of saying “Newtonian physics” because Isaac Newton was the guy who figured this all out back around 1700 AD. Descartes (1596-1650) investigated these ideas, and he came up with the notion of momentum. Galileo (1564-1642) also worked on these concepts, and he came up with the law of the conservation of momentum. Isaac Newton (1643-1727) continued where these two left off, and came up with three laws of motion. Gottfried Leibniz (1646-1716) played around with Newton's Laws and discovered another law we call the conservation of energy. Together, the conservation of momentum, the conservation of energy, and Newton's three laws of motion are the basis for classical mechanics or Newtonian physics, whatever you want to call it.

Basic Units of Measurement

The base units of measurements in physics are mass, distance, and time. From there, we can derive all other units. The SI units for mass is kilograms, distance is meters, and time is seconds.

Derived units: Momentum

From the basic units, we can define some derived units. The unit I want to start with is called momentum. Momentum is defined as the mass of an object multiplied by its velocity. The symbol "p" represents momentum.

Momentum (p) = mass (kilograms) * velocity (meters per second)

A change in momentum is called an Impulse. It also has the units of kilograms*meters/second.

Impulse = change in momentum(p) = mass (kilograms) * velocity (meters/second)

Side note: Any value that represents a difference (or change) always has the same units as the thing it is a difference between. The difference between 7 meters and 4 meters is 3 *meters*.

Impulse is a change in momentum. If you have a mass with a certain momentum, you can change it's momentum by having it collide with another mass with a different momentum. Billiard balls colliding on a pool table, hockey pucks on ice, and similar scenarios are all situations that are exercises in momentum being moved around from one mass to another. Another way to change momentum of a mass is to carry extra mass along with you and then eject it in the opposite direction you want to go. Common scenarios in physics classes involving this sort of momentum transfer include rockets, cannons, and firearms.

Suggested Unit Name for Momentum

There is no SI unit name for momentum. It is instead listed as kg*m/s. This is actually rather cumbersome and nonintuitive. I suggest that the unofficial SI unit for a change in momentum, i.e. impulse, be called a "shot".

1 unit of impulse = 1 ***shot*** = 1 kg * 1 meter / 1 second

This actually takes advantage of an intuitive understanding of impulse and momentum, namely that momentum can be generated by shooting a projectile away from the object whose momentum you want to change. And 1 "shot" of momentum is 1 kg*m/sec.

Newton's First Law, the Law of Inertia

Newton's first law states that a body at rest will stay at rest, and a body in motion will stay in motion with the same speed and direction, unless some outside force changes it.

This is sometimes called the Law of Inertia. But inertia is just another way of saying momentum.

This notion of momentum can be counterintuitive to our interpretation of normal day-to-day experiences. Throw a ball and it eventually comes to a stop. Slide down a hill of snow on a sled and you may be moving pretty fast at the bottom, but you will eventually come to a halt. Everything we do in our day-to-day experience tells us that a body in motion eventually comes to a halt.

Newton's first law, the law of inertia, doesn't violate your day-to-day experience. It's just that in your day-to-day experience, almost all motion involves an invisible external force that acts only on objects in motion, not objects at rest, and that invisible external force always works in the direction opposite the direction of motion, which always has the effect of slowing the object down until it comes to a halt. And that force is called friction.

So, while “an object in motion will eventually come to a rest” might get you through your day-to-day experience of the world, I promise you it will trip you up in your quest to understand classical physics. If you want to have an easier time learning this stuff, replace “an object in motion will come to a rest” with “an object in motion stays in motion until some external force changes it” and then remember that friction is one external force that shows up almost everywhere in your day to day life.

Law of Inertia, Intuitive Example

Air hockey tables are great ways to play around with classical physics. If you need an excuse to get an air hockey table, just tell people you're doing “research in the field of Newtonian physics”.

With an air hockey table, we can reduce the effects of friction that we can see. This allows us to demonstrate in the real world, with fairly common items, a good approximation of Newton's first law, that an object in motion will stay in motion. Simply turn on your air hockey table, put the puck on the table, and give it a nudge. You will see the puck travel a considerable distance at a constant velocity before it's velocity will slow enough to be noticeable.

This is a demonstration of Newton's first law. An object in motion will stay in motion unless an external force acts on it.

Law of Conservation of Momentum

There is another law relating to momentum called the conservation of momentum. It's not exactly the same as Newton's first law, the law of inertia. Newton's first law states that given one object with some velocity (zero or non-zero) and no external forces acting on that object, the object's momentum will not change. But if momentum of one object will not change, then we can say it is “conserved”.

And if its true that the momentum of one object is conserved, we can intuit that its true for two or more objects as well. Which means that if we have two objects moving in zero friction and no external forces act on those two objects, the sum total of the momentum of those two objects will remain the same. This means that if the two objects collide, their total momentum after the collision must be equal to their total momentum before the collision.

Conservation of Momentum Intuitive Examples

You see the law of the conservation of momentum being followed in an everyday scenario if you ever play billiards or pool. Say the only ball left to sink is the eight-ball. Say you have to do a straight line shot from cue ball to eight ball to corner pocket You hit the cue ball and give it some momentum. Right after you hit the cue ball, the cue ball starts moving and all the billiard balls on the table become a closed system. No other external forces act on the balls. You'll see the cue ball move towards the eight-ball at some velocity. And if you lined up your shot just right, when the two balls hit, the cue ball will come to a stop and the eight-ball will start moving at the exact same speed as the cue ball was moving, until it goes into the corner pocket.

Momentum within the system of the billiard balls is conserved. Whatever the momentum of all the balls was before the collision, the balls will have that same total momentum after the collision.

Another simple everyday example of conservation to momentum is one of those toys that has a series of steel marbles suspended from strings and forming a line. The device is called, appropriately, “Newton's Cradle”. If you pull one marble up and away from the rest, when it collides with the other marbles, momentum must be conserved, and one marble on the opposite end will go flying up while the others, including the one that was originally moving, remains stationary. If you pull up two marbles and release them, then when they collide with the other marbles, two marbles on the opposite end will go flying.

The conservation of momentum states that in any closed system (no external forces working on our masses), the momentum of the system remains constant, even through collisions.

*Newtons Second Law: Force = Mass * Acceleration*

With Newton's first law, we know that an object with a fixed mass and a fixed velocity (zero or positive) will maintain that velocity until something acts on it to change its velocity (either its speed, or direction, or both). So, we have objects that can move at constant velocity, and if they collide, we can see velocities of objects change so that momentum is always conserved.

But how do we get an object moving in the first place? Newton's second law says we need to apply a force to an object. A force is a vector quantity that has a magnitude and direction. You stand on something and push with a force of your weight in a downward direction for as long as you stand on it. You can use a cue stick to hit a cue ball with a large force for a very short period of time. You can use a rope attached to some object and pull on that rope for as long as you want to apply a force to the object.

We will explore exactly how we define a force and measure it and so on later, but the basis of Newton's second law is simply that if you push on something, that something will start increasing its velocity.

Fairly intuitive, right? Well, as soon as you apply a force, newton realized that the force cannot exist alone. It must have an opposite somewhere. And this is his third law of motion.

Newton's Third Law: Every Force has a -Force

When you hit a cue ball with a cue stick, the stick puts a large force on the cue ball for a very brief moment. But this requires that the cue ball put a large force on the cue stick, in the exact opposite direction, for an equally brief period.

When you attach a rope to an object and pull on the rope, you put a force on the object. But the object will put a force on the rope and therefore on you in the opposite direction.

When you stand on earth and feel gravity pull you downward, the earth will feel the exact same force pulling it upward. A gravitational force can only exist between two masses. It cannot exist as a force if only one mass exists.

In our day to day life, Newton's Third Law may seem counterintuitive. When you push on an object, the object moves, you don't. When you jump up and down, you feel the gravitational force of the earth pulling you down, but you don't feel the earth being pulled up. When you hit a billiard ball with a cue stick, the ball moves, you don't. Much of the non-intuitiveness of this opposing force idea comes from the fact that the everyday forces we interact with have the opposing force act on what is essentially an immovable object to us: the Earth. The other source of confusion is that we are often dealing with forces that are transferred through friction into the earth. We push on an object, friction between our feet and the ground prevent our feet from slipping, and the opposing force ends up pushing back on the ground, which doesn't move in any measurable way.

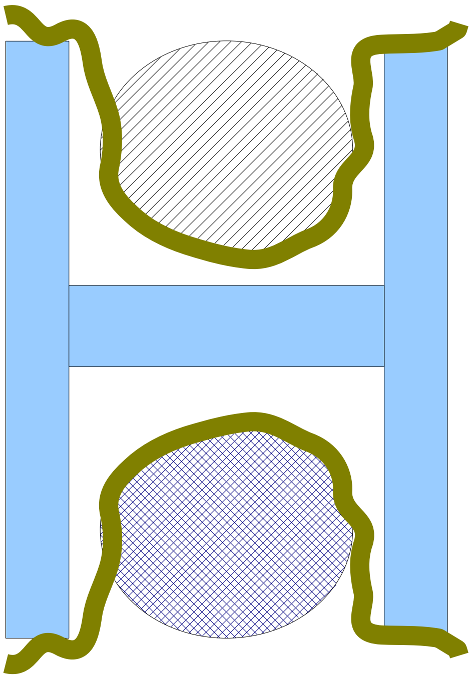
Lets look at some more basic examples involving more manageable sizes of masses and little friction.

Newton's Laws: Intuitive Example Air Hockey

Get your air hockey table cleared off. You'll need some pieces of wood and rubber bands. Make an "H" shaped frame with the wood. Build it so the two vertical pieces are just wide enough apart that a puck can fit in between them. The frame should be light enough that it "floats" like a puck when the air hockey table is turned on. We don't want friction to mess up our test results. If it's too heavy to float, then put some "feet" underneath the frame that sticks out to the sides. Leave the area where the hockey pucks go open so the pucks will float too.

Then find some nice, fat, wide rubber bands and attach them to the ends of the "H" frame.

Here's our air hockey puck shooter:



Then put the whole contraption on the air hockey table, put the two pucks in the frame, pull them back against the rubber bands, and then hold them for a moment to prepare yourself. You want to pull the pucks back against the rubber bands and then push down on the pucks and the H frame to hold everything in place. Then take a deep breath and let go of everything at the same time.

If you built everything just right, i.e. both rubber bands have the same amount of tension on them, both are pulled back the same amount, both pucks are released at exactly the same time, and so on, then what should happen is that both pucks will shoot off in opposite directions at exactly the same speed, and the "H" frame shouldn't move at all, staying exactly where it was before you let go.

This may not seem a very important demonstration, but it shows Newtons Third Law. To put a force on one puck to accelerate it, there is a force in the opposite direction. In day-to-day experiences, there is often a lot of friction involved in pushing on an object that the opposing force doesn't have any visible effect.

You pull on a rope attached to some object, and the object moves, but you're standing on ground with boots with good traction, so you don't move. If you and the object were on roller skates, you'd see that removing friction completely alters our experience of the world.

The H-frame rubber band powered puck shooter example actually demonstrates all three of Newton's laws.

Newton's first law: An object at rest remains at rest unless a force acts on it to accelerate it. Put a puck on the table by itself with no H-frame and no rubber bands, and that puck will basically remain wherever it is. If it is moving with some velocity, it will maintain that velocity.

Newton's second law: A force on an object will accelerate an object. The H-frame and rubber bands will put a force on the puck and accelerate the puck.

Newton's third law: For every action, there is an equal and opposite reaction. To apply a force to the puck, the puck will apply a force to the H-Frame. If you do the experiment with only one puck, then when you release the rubber band, the H-frame will push on the puck and the puck will push on the H-frame. The puck will move forward and the H-frame will move backward.

Conservation of Momentum: The only way to keep the H-Frame from moving is to put another puck in the other side of the H-frame and accelerate the second puck with an equal amount of momentum in the opposite direction. If both pucks are the same mass, and both rubber bands apply the same force in opposite directions, then both pucks should have the same momentum in opposite directions, and the total momentum of the system should remain balanced, and the H-frame won't have to move.

Without the second puck, or with unbalanced pucks or unequal rubber bands, the H-frame will always move because momentum must be conserved.

Newton's Laws of Motion: Roller Skating Tug Of War

Fanty and Mingo are identical twins. (They are exactly identical, but if anyone asks, Fanty is prettier.) Fanty and Mingo decide to go roller skating for the first time. Fanty figures out how to move himself forward on the roller skates, but Mingo is having a harder time learning how to do it. All Mingo does is fall down as soon as he tries to move his feet. Fanty gets a rope and gives one end to Mingo. Then Fanty moves to the other end of the rope and stands in front of Mingo and tells Mingo that he will pull the rope and pull Mingo forward. What happens when Fanty pulls the rope?

Fanty pulls the rope and puts a force on Mingo to move Mingo forward. But the force on Mingo forward puts an opposite force on Fanty to move backward, towards Mingo. Since Force = Mass * Acceleration, and Fanty and Mingo weigh exactly the same amount, they will both accelerate at exactly the same rate, but in opposite directions, i.e. towards one another.

If they have 10 meters of rope, Mingo will move 5 meters forward and Fanty will move 5 meters backwards, and then they'll both be standing next to each other. They decide to try again and make it a tug of war contest.

Every time they have a tug of war on roller skates, Fanty and Mingo tie.

Seeing this, a couple of the Hong Kong Cavaliers decide to get in on the action. Rawhide and New Jersey put on some skates and put 10 meters of rope between them. Rawhide puts a force on the rope to pull Jersey forward, and this puts a force on Rawhide backwards. What happens?

Jersey weighs a lot less than Rawhide. When Rawhide pulls on the rope, it puts the same force on Jersey as it does on Rawhide (just in opposite directions). Using Force = Mass * Acceleration again, we see that putting the same force on different masses will make the lighter mass accelerate faster. So Jersey moves forward faster than Rawhide moves backward.

Every time Rawhide and Jersey have a tug of war on roller skates, Rawhide wins.

You can try something like this on your air hockey table. You'll need some way to put a rope between two pucks and reel it in. A very long, very weak rubber band would be one way to do it. Attach the rubber band to the center of two pucks, hold the pucks at opposite ends of the table, and then release them at the same time. They should hit each other in the center of the table. If you double the weight of one puck and repeat the experiment, the pucks should collide on the half of the table closer to the heavier puck.

If you want to try this with an actual tug of war, you'll need one of those tiny, tiny remote control cars and some thread. Mount the car upside down on one puck so you can use it as a winch. If the puck doesn't float with the extra weight, make the base larger. Then attach the thread to the axle of the car and to the other puck. Use the remote control to reel in the thread and control how fast you reel it in. If both pucks weigh the same, they should meet at the center of the table. If one puck is heavier, the cars will meet on the half of the table closest to the heavier puck.

Some Observations from the Example

Note that it doesn't matter which puck is doing the “pulling”. One puck has a car acting as a winch, but that will put the same force on both pucks, just in opposite directions. This is Newton's Third Law again. It is not the puck with the winch that will win the tug of war contest, it is the puck that is heaviest that will win.

Also note that it doesn't matter if the experiment has two masses being pushed apart (like the H-Frame and rubber band experiment) or two masses being pulled together (like the roller skate tug of war experiment), all three of Newton's laws apply and are followed. The force is equal to the mass times acceleration, and moving any object creates an equal and opposite reaction.

This is another way of saying that conservation of momentum is always observed and followed. Whatever experiment we used, the momentum of the system before we started was zero, and the momentum of the system after the rubber bands were released was also zero, which was accomplished by having two masses moving in opposite directions so that the total momentum of the system was still zero.

Lastly, the rubber bands and winches and so on in these experiments are ways of doing work on the system, which alters the energy of the system as a whole. We will get to work and energy definitions soon, but the point here is that doing work on a system doesn't change the conservation of momentum requirement of the system. Adding energy to the system doesn't change the overall momentum of the system. Our H-frame, rubber band powered, puck shooter added energy to the system, but the overall momentum before and after remained the same.

Example: Firing One Bullet

In the tug of war on roller skates example, we saw that whoever is heavier will win the tug of war. We saw this using the H-frame puck shooter too, using two pucks of equal mass, and then trying only one puck and seeing the H-frame recoil.

So, what happens when something really heavy shoots something much lighter than it?

Example: You have a cannon weighing 100 kg. It shoots a cannonball weighing 1 kg at a velocity of 100 meters per second. The momentum of the cannonball is

momentum (p) of cannonball = mass * velocity

momentum of cannonball = 1 kg*100m/s = 100 kg*m/s.

Conservation of momentum says that the momentum of the cannon after the shot must also be 100 kg*m/s, but in the opposite direction.

Momentum of cannon = 100 kg*m/s

And we also know the momentum of the cannon is the mass of the cannon multiplied by its recoil velocity.

Momentum of cannon = 100 kg*m/s = mass of cannon * velocity of cannon

And we know the mass of the cannon is 100 kg.

100 kg*m/s = 100 kg * velocity of cannon

Solving for velocity

velocity of cannon = 100 kg*m/s / 100 kg = 1 m/s

The cannon will recoil at a velocity of 1 meter per second until friction and other forces finally brings it to a halt.

Cannon on a Cart

So we showed what happens when a 100 kg cannon fires a 1 kg cannonball at 100 m/s. Imagine what would happen if we took the cannon and a bunch of cannonballs and put them all on a cart and started shooting off a series of cannonballs, one after another. What happens?

Say we've got 1 cannon and 10 cannonballs. Say the cart itself weighs another 100 kg. And say the person running the cannon, the cannoneer, weighs 100 kg. So, we start out with the mass of the cart, the the mass of the cannon, the mass of the cannoneer, and the mass of ten cannonballs, i.e. 100+100++100+10=320kg. Our cannoneer takes the cart out to the Salt Flats, loads up the cart, and then fires the cannon.

What happens?

The momentum of the cannonball must equal the momentum of the rest of the platform (cart, cannon, and cannoneer) in the opposite direction.

Mass of cannonball * velocity of cannonball = mass of platform * velocity of platform.

We already know how fast our cannon shoots a cannonball. 100 m/s. And we know how much a cannonball weighs. 1 kg.

1 kg * 100 m/s = 319 kg * v

Solving for V:

v = 1 kg * 100 m/s / 319 kg = .313 m/s

So the platform with the cart, the cannon, the cannoneer, and the remaining cannonballs is rolling at 0.313 meters per second.

It's a very low friction cart and the Salt Flats are very flat, so once we start rolling at 0.313 meters per second, conservation of momentum and a lack of any noticeable friction means we keep rolling at 0.313 meters per second while the cannoneer reloads. Then the cannoneer fires another cannonball. What happens?

The momentum of the second fired cannonball must equal the change in momentum of what's left of the platform.

Cannonball mass * cannonball velocity = platform mass * change in platform velocity

1 kg * 100m/s = 318 kg * deltaV

deltaV = 1 kg * 100m/s / 318 kg = 0.314 m/s

So the velocity of the cannon platform will increase by 0.314 m/s. It was already 0.313 m/s, so after the second shot, the platform is moving at 0.627 m/s.

The cannoneer reloads and fires another shot. What happens?

Again, the momentum of the cannonball must equal the change in momentum to the platform.

Cannonball mass * cannonball velocity = platform mass * change in platform velocity

1 kg * 100m/s = 317kg * deltaV

deltaV = 1 kg * 100m/s / 317 kg = 0.315 m/s

Firing the third cannonball increases the velocity of the platform by 0.315 m/s. The platform was already moving at 0.627 m/s, so after the third cannonball, it is moving at 0.942 m/s.

Did you notice a pattern?

First shot adds 0.313 m/s to platform, final velocity is 0.313 m/s

Second shot adds 0.314 m/s to platform, final velocity is 0.627 m/s

Third shot adds 0.315 m/s to platform, final velocity is 0.942 m/s

Every shot, the platform changes velocity by a little bit more than it changed for the previous shot. Why? Because the platform is getting lighter every shot. A cannonball weighs 1 kilogram, so every shot reduces the mass of the platform by 1 kg. But every shot is a 1 kg cannonball moving at 100 m/s, which is always the same amount of impulse. Adding the same impulse to an object that decreases in mass will have the change in velocity per impulse always be a little bit more that the previous.

This is something that rocket designers have to deal with. Rockets work by creating chemical reactions which “shoot” gas “cannonballs” backwards and push the rocket forward. But the mass of the rocket changes as it burns fuel. Each “shot” makes the rocket a little bit lighter.

Back to our cannonball platform, we could reduce the mass of the cannonball so that the mass of the platform doesn't change as much per shot. Then we could compensate for a lighter cannonball by increasing the muzzle velocity of the cannonball so that it has the same impulse per shot.

If the cannonball weighed 0.1 kg and had a muzzle velocity of 1000 m/s, it would have the same impulse, but the mass of the cart would change less per shot. If the “cannonball” weighed 1 gram (0.001 kg) and the muzzle velocity was increased to 100,000 m/s, then it would still have the same impulse per shot, and a change of 1 gram wouldn't be very noticeable for several shots.

The Pshooter Cannon on a Cart

Imagine we have a cannon that shoots a cannonball that has so little mass that we can fire a bunch of shots without noticing any change in mass to our platform. And imagine that our cannon shoots these cannonballs at such high velocity that we can measure a noticeable change in velocity per shot.

We defined a “shot” as a change in momentum, an impulse.

1 change in momentum = 1 unit of impulse = 1 ***shot*** = 1 kg * 1 meter / 1 second

And one “shot” is like our cannon firing a shot on the platform. The only change we want to make is to imagine that we use really, really, really small cannonballs fired at really, really, really, high velocities so that we still have a noticeable change in momentum, a noticeable impulse, per shot fired, without affecting the overall mass of the platform.

We'll call our imaginary cannon a pshooter. The symbol for momentum is “p” and “shot” is 1 kg*m/s of momentum, so “pshooter” is something that shoots 1 kg*m/s of momentum every second. And just because this is an imaginary cannon, we're going to make the cannon have zero mass, just to make our calculations easier. We'll then be able to put our pshooter cannon on a platform and not have to worry about the mass of the pshooter, only the mass of the platform.

So, imagine a platform that weighs 1 kilogram and put one pshooter on it. Then imagine shooting the pshooter several times, what happens?

We start with the platform at rest.

We fire a shot, which creates an impulse of 1 kg*m/s which changes the momentum of our platform by 1 kg*m/s. So our platform is traveling at 1 m/s.

A second later, we fire another shot. What happens? The shot creates another impulse of 1 kg*m/s, which changes the momentum of the platform by the same amount, so our platform changes velocity by 1 m/s. It was already traveling at 1m/s, so now it is traveling at 2 m/s.

A second later, we fire another shot. The velocity of the platform increases by 1 m/s, so the platform is traveling at 3 m/s.

Here is a table after our pshooter fires a shot every second for a few seconds.

time	velocity
0	0
1	1
2	2
3	3
4	4

So, our pshooter is firing a shot every second.

1 pshooter = 1 shot per second

A shot is 1 kg * m/s

1 pshooter = 1 (kg * m / s) / s

Grouping the “seconds” together:

1 pshooter = 1 kg * m / s^2

And meters/s^2 is meters per second per second, which is just another way of saying acceleration.

1 pshooter = 1 kg * acceleration

And when we look at what happens after each shot, we see our velocity increase at a linear rate per shot, which is the same as acceleration.

If you think this formula looks familiar, you're right. Remember Newton's Second Law?

Force = mass * acceleration

That's exactly what we have here. When we put a pshooter on a mass, it moves that mass at some fixed acceleration.

And this motion is the result of some fixed “force” on the mass. What is a force? It's our next derived unit.

Note those so interested can go to the back of this document and find a perl script which will calculate velocities and distances traveled for a mass having some number of shots per second applied to it. Calculating one shot every second doesn't actually give us smooth acceleration, it accelerates us during the shot, and then our velocity is fixed until the next shot. The shot-per-second perl script breaks the calculation up into as small of a slice in time to show that if you distribute a single shot out over the entire second, you approach fixed, and smooth, acceleration.

Derived Unit: Force = Newtons

Newton's Second Law defined “force” for us.

force = mass * acceleration

Inserting SI units for mass and acceleration:

force = mass (kg) * acceleration (m/s^2)

Which gives us the units for force:

force (kg * m/s^2) = mass (kg) * acceleration (m/s^2)

A force of 1 kg * m/s^2 is called a “newton” in SI units.

1 newton of force = 1 kilogram * meter / second^2

A “shot” is 1 kilogram*meter/second. And our “pshooter” fired 1 shot per second.

1 newton of force = 1 kg * (m/s^2) = 1 shot / second

Our pshooter, firing 1 “shot” per second, exerts one newton of force on an object.

The pshooter

There are many ways to intuit what a force is. A mass moving at a certain acceleration is one way. For example, the pull on a mass exerted by the earth's gravity is a force of newtons. Earth's gravity is 9.8 m/sec^2. So, a mass of about 1/10 of a kilogram in your hand will exert a force downward on your hand of about 1 Newton. A deck of standard playing cards weighs about 100 grams, so the downward force it exerts on Earth would be about 1 newton.

Another way to think of force is a little pshooter. If you can imagine a virtual gun firing little masses at a constant rate, giving them momentum in one direction and exerting a force in the other direction, then that is the “pshooter”. That is another way of visualizing a force. The pshooter might help you keep in mind Newton's Third Law, that to create a force in one direction, we must have an opposing force.

1 shot = 1 kilogram * meter / second
1 pshooter = 1 shot / second = 1 kg*m/s^2 = 1 Newton

For the earthbound, force can be thought of as gravity exerting some force vertically or a pshooter exerting some force horizontally.

Quick Review: Proving Newton's Second Law

The last few sections showed us the calculations to prove Newton's Second Law

force = mass * acceleration

And we proved this using nothing but Newton's first law, the conservation of momentum. We were able to calculate a pshooter adding an impulse of momentum to some mass every second, and we showed that the mass will move at a fixed acceleration.

Let's have some fun with our pshooters cannons.

The Pshooter Races

Our identical twins, Fanty and Mingo, have identical skateboards and the both get identical pshooters that exert 1 shot per second of force. They decide to have a race. Both of them get on their skateboards and aim their pshooters back and start accelerating forward.

Who wins the race?

Both pshooters exert the same force, and both Fanty and Mingo weigh exactly the same. With the formula force = mass * acceleration, it isn't too hard to see that both will accelerate at exactly the same rate, and the race will always be a tie.

Fanty and Mingo always tie.

Seeing this, some Hong Kong Cavaliers decide to get in on the action. Rawhide and Perfect Tommy get identical pshooters and identical skateboards and have their own race. They both aim their pshooters back and start accelerating towards the finish line.

Who wins the race?

Rawhide weighs more than Perfect Tommy. Both pshooters exert the same force. So, given force = mass * acceleration, we can see that the person with the least amount of mass will accelerate faster. So, Tommy wins. Cause he weighs less than Rawhide. And cause he's perfect.

Perfect Tommy always wins against Rawhide.

Seeing this, Professor Hikita wonders if it would be possible to design a skateboard in which every race is a tie, no matter who drives it. So he starts by building a special skateboard powered by pshooters. Using Force = mass * acceleration again, he realizes that if he puts a weight sensor in the skateboard, he can turn on one pshooter for every kilogram the driver weighs, and then the skateboard will always get the exact same acceleration regardless of who is driving.

Driver 1 weighs 100 kg, skateboard turns on 100 pshooters. Driver 2 weighs 75 kg, skateboard turns on 75 pshooters.

Force = mass * acceleration

solve for acceleration

acceleration = force / mass

Force is a number of pshooters. A pshooter is nothing more than a "shot" (kg*m/s) per second.

Acceleration = pshooters / mass

Calculate for a 100 kg driver, Hikita's skateboard automatically turns on 100 pshooters

acceleration = 100 pshooters / 100 kg driver = 1 m/s^2

Calculate for a 75 kg driver, Hikita's skateboard automatically turns on 75 pshooters

acceleration = 75 pshooters / 75 kg driver = 1 m/s^2

So, Hikita's skateboard accelerates at 1 m/s^2 no matter who is on it.

Buckaroo gets back from saving the planet with his rocket car and sees the Cavaliers playing around. He takes all of the skateboards and rewires them so that they turn on 10 pshooters for every kilogram that the driver weighs.

Reno tries the skateboard. He weighs 78 kg. What is his acceleration?

Acceleration = 780 pshooters / 78 kg driver = 10 m/s^2

Jersey tries the skateboard. He weighs 67 kg. What is his acceleration?

Acceleration = 670 pshooters / 67 kg driver = 10 m/s^2

No matter who drives Buckaroo's modified skateboards, they all accelerate at 10 meters per second per second.

Artie sees all this going on and is a little bit ticked off. He doesn't care if everyone has the same acceleration on a skateboard, cause he doesn't see what difference it makes in the real world.

Buckaroo tells Artie that the skateboard is a good model for Earth's gravity. Artie then shrugs and says he just wants to hear some good music out of them.

Do you know that girl?

There is a scene from Serenity I really like. Mal is at a bar meeting with Fanty and Mingo. They're discussing business, when out of the blue River Tam, a young woman who has been on Mal's ship for months at this point, starts beating up everyone in the bar. Fanty sees this and asks Mal a question.

> Fanty: Do you know that girl?
> Mal: I really don't.

Mal had only seen River as a quiet, somewhat crazy, girl on his ship who seemed scared of her own shadow and kept to herself mostly. And he'd seen her this way for months. When he sees this little girl beating up everyone in the bar (and doing an really good job of it I might add) Mal realizes he has no clue who River really is.

Moments later, River's brother, Simon, comes in and says some safe word which causes River to pass out.

If you want to “know” gravity, if you want to know it from the inside out, you'll have to look elsewhere. All I can tell you is the safe word which will help you get it to do what you want to do. I don't know why it works that way. It just know it works.

What is Gravity?

If you imagine that all mass exerts some force that attracts all other mass to it, you've got the basic idea of gravity. That's as good a starting point as any.

Imagine two billiard balls, out in deep space, away from any external influences, away from any friction, away from any injection of energy. If you placed a spring between the two billiard balls, the gravity between them would eventually pull the two billiard balls towards each other and compress the spring some amount before it reached an equilibrium point and stopped.

The formula to calculate the amount of gravitational force exerted between two masses follows:

$$\text{Force (newtons)} = \text{grav\_const} * \frac{\text{mass of first object (kg)} * \text{mass of second object (kg)}}{\text{distance between objects \^{}2 (meters)}}$$

The gravitational constant is approximately 6.7e-11, which is a really, really small number.

If you have two billiard balls in deep space, and each one weighs 1 kilogram, and you put them 1 meter apart, and have some spring to measure the force between them, it would measure .000000000007 newtons of force. If you're using a spring, it will compress a really, really, really small amount.

Mass, it turns out, puts pshooters on any mass around it. The number of pshooters depends on how much mass is involved and how much distance is between it. In our 1 kg billiard ball in deep space example, each mass puts 0.00000000007 shots per second of force on each other. Which isn't much.

But in our everyday circumstances, we're on earth, fairly close to sea level. And in that scenario, you've got enough mass in planet earth that 0.0000000007 pshooters per kilogram adds up to something you can actually feel.

The Earth as a Billiard Ball

So, lets change the billiard balls in space example. Instead of 2 billiard balls, each with a mass of 1 kg, 1 meter apart, lets supersize it. Lets have two billiard balls, one weighs 1 kilogram, and one weighs about the mass of the earth, or about 6e24 kilograms. Now, for the radius, lets put our two billiard balls about 6400 kilometers apart (6.4e6 meters). Why six thousand kilometers? Because that's about the distance from the center of the Earth to its surface.

So here is the calculation:

Force = G * (m*m) / r^2
f = 6.7e-11 * (1 * 6e24) / (6.4e6^2)
f = 6.7e-11 * 6e24 / 4.1e13
f = 4.0e14 / 4.1e13
f = 9.7 m/s^2
f =~ 10 m/s^2 = 10 newtons = 10 pshooters per kilogram

Standard gravity is actually 9.8 m/s^2, so we came out pretty close considering we only used 2 digits of accuracy in our calculation.

Looking at the formula, we can see that if we change the 1 kilogram mass to 2 kilograms, the force doubles. A 1 kilogram mass will feel about 10 newtons of force. A 2 kilogram mass will feel about 20 newtons of force. 3 kg->30 newtons.

A 1 kg mass will act as if 10 pshooters are pushing it down. A 2 kg mass will act as if 20 pshooters are pushing down on it. A 3 kg mass will act as if 30 pshooters are pushing down on it.

Which is a lot like Buckaroo's skateboard, eh?

The Earth as Buckaroo's Skateboard

The earth exerts a gravitational force of nearly 10 pshooters on every kilogram of mass near its surface.

This is similar in function to what Buckaroo's skateboard did. Buckaroo's skateboard had a scale built into it which determined the weight of the driver. And then the skateboard would turn on 10 pshooters for every kilogram the driver weighed. (ok, and the Earth's gravitational pull is actually 9.8 pshooters, but you can approximate it as 10.)

The only difference is that Earth's gravity pulls down and Buckaroo's skateboard moved horizontally.

And we already showed what happened when someone got on Buckaroo's skateboard. Remember? Everyone tied. It didn't matter how much you weighed, you got on Buckaroo's skateboard and you would accelerate at 10 meters per second per second.

Well, that's what gravity does. When you have a “race” between two objects falling near the Earth's surface, it doesn't matter how much they weigh, because the earth puts more pshooters on the more massive objects, and everything falls at the same rate of acceleration: 9.8 meters per second per second.

If you have two masses, one is 30 kg and the other is 700 kg, and release them at the same time from the same height, they will both hit the ground at the same time.

Again, the mass is irrelevant in the “race” because each mass gets a different number of pshooters on it based on its total mass. Like Buckaroo's skateboard, the force of gravity is proportional to the mass being dropped, so the acceleration is constant regardless of the mass.

And the reason the force of gravity is proportional to the mass being dropped goes back to the original idea of gravity:

> every kilogram of mass exerts a force on every other kilogram of mass

And all those forces add up together. So the more mass, the more the force. Gravitational force becomes directly proportional to the mass.

So, it acts like Buckaroo's skateboard. The difference being that Buckaroo's skateboard had to use some software and an electronic scale and other things to make sure the number of pshooters it turned on was proportional to the weight of the driver, but the Earth just does it naturally.

Why masses have forces of gravitational attraction between them, I have no idea.

This is where you get to the difference between Simon knowing the code word for River, but no one really knows what happened to River or who she is on the inside. Do you know that girl? I really don't.

But you really don't have to understand how Buckaroo's skateboard works down to the last detail to ride it. And from that perspective, you should now have a sense of how gravity works. If anyone gets on the skateboard, they accelerate at the save value, about ten meters per second per second.

Rest Stop: Gravity is Pshooters per Kilogram

Still with me, I hope? If not, you might want to go back to where you got lost, back up to the previous section where you weren't lost, and start reading again from there. We're about to shift gears to some new topics and if you're lost, you'll only become more lost.

The summary of the last section about gravity is that basically you can think of gravity as putting some number of pshooters on every kilogram of mass it affects. The number of pshooters depends on the number of kilograms (and eventually, the distance between the masses). In the case of the earth, its about 10 pshooters per kilogram.

Force of gravity on earth = 9.8 pshooters per kilogram

Some other things to keep in mind about gravity is that it is a positive force only and only exists between masses. If you could somehow have an isolated mass by itself, it wouldn't feel any gravitational force because gravity is a force exerted between masses, not by a single, lone, mass. Gravity is Nature's way of observing Newton's Third Law. For every force, there is an opposing force. For the earth to put a 10 newton force on an object, the object will put a 10 newton force on the earth.

Because of Newton's Second Law, Force = mass * acceleration, the small object does most of the moving, and the earth doesn't hardly move at all.

And because gravity is a force that is always proportional to the mass of the object, any two objects at the same height on earth will fall at the same rate.

Of course, a light object with air friction may be slowed down by the air friction, but that's friction coming into our experiments and messing us up again, right? If you have two objects that are heavy enough and both have the same air friction affecting them, then you'll seem them fall at essentially the same acceleration.

OK? Do you have a grasp of gravity? Then on to the next derived unit.

Derived Unit: Work = Force applied over Distance = Joules

Our next derived unit is called “work”. Work is defined as applying a force over a distance. To do work in physics, you must move something. You can't just push on a wall and not have the wall move. That may exert your muscles, but it doesn't qualify as “work” in the physics sense of the word. The only part of the force that qualifies for work is the part that is in the direction of the movement.

If you apply a force at a direction 45 degrees from the ground and push on a cart and move the cart 1 meter horizontally, then the part of the force that did work was the horizontal component, and you might have to do some extra math to figure out the horizontal component.

The SI unit for work is a Joule. It is measured as a force over a distance. And since we know that a force is shots per second, and a shot is kg*meter/second, that means that a joule would be shots per second over a distance or:

1 joule = force * distance
1 joule = (shots per second) * distance
1 joule = [(kg*m/s) / second] * m
1 joule = kg m^2 / s^2

When you compress it all together, it's a very strange jumble of units. But if you remember its a force over a distance, and force is shots per second, then that might help.

Alternatively, a joule is a force over a distance, and force can be defined as mass*acceleration.

1 joule = force * distance
1 joule = (mass*acceleration) * distance
1 joule = kg * (m/s^2) * m

Work Lifting Weights

As we mentioned earlier, the earth's gravity imparts a constant acceleration on all masses. The earth's gravity creates an acceleration of 9.8 meters/second^2 on a mass. Using Force=mass*acceleration, we get that a force of 1 newton would be exerted on your hand if you held a mass of about 100 grams.

A deck of standard playing cards weighs about 100 grams. Now, start by putting the deck of cards on the floor. Then pick the deck up and put it on a table (which should be about 1 meter off the floor).

Congratulations, you just did 1 joule of work. You exerted 1 newton of force on the cards to lift them out of gravity's pull, and you did so for a distance of 1 meter. So, 1 joule of work.

Since gravity imparts a fixed acceleration on any mass, the work done moving that mass up is defined using the following translation:

> 1 joule = force * distance
> force = m*a
> 1 joule = (mass*acceleration) * distance

So, the work to lift an object straight up is equal to the mass of the object in kilograms multiplied by the earth's gravitational constant (9.8 m/s^2) multiplied by the distance you raise it in meters.

> work (lifting mass vertically in gravity) = mass(kg) * gravity (m/s^2) * height (meters)

The short version is

> vertical work = mass * gravity * height

You must do positive work to lift a mass. You can recover work if you lower a mass. For example, water wheels recover work by lowering water.

Note that work is not a function of how fast you lift the deck of cards. How fast you lift the cards is a function of something called “power”. We'll discuss power later.

Work Pushing Mass With No Friction

Another way to do work is to exert a force on a mass over a distance while the force causes the mass to accelerate. Such as putting some pshooters on a skateboard to move the skateboard.

> work = force * distance

The distance traveled under constant acceleration is given by

distance = ½ * acceleration * time^2

And force is “shots per second”

work = (shots/second) * (½ * acceleration * time^2)

And a shot is a kilogram*meter/second.

work = kilogram*meter/sec^2 * ½ *acceleration * sec^2

cancel the sec^2

work = kilogram * meter * ½ * acceleration

acceleration is meter/sec^2

work = kilogram * meter * ½ * meter / sec^2

rearranging the order a bit, and grouping two “meters” together

work = ½ * kilograms * meters^2 / sec^2

meters^2 / sec^2 is the same as (meters/sec)^2

work = ½ * kilograms * (meters/second)^2

and meters per second is just velocity

work = ½ * kilograms * velocity^2

Whoah, there. Did you get that? We start with the definition of work being force times distance, and we show that the work it takes to accelerate some mass from zero to some velocity is a function of the mass and the square of the velocity.

That's actually somewhat non-intuitive. You might want to go through the steps just to make sure I didn't try to pull a fast one on you somewhere in the equations.

Derived Unit: Energy = the ability to do work = Joules

We have another derived unit called “energy”. Energy is defined as the ability to do work. The ability to exert a force over a distance. Energy can store its ability to do work in some form until we decide we want to do some work, at which point, we convert some energy to work, and lower our energy.

There is a law of conservation of energy which says that the energy of a closed system remains constant. This means that it energy cannot be created or destroyed. It can, however, change forms. The work we do, therefore, reflects the change in energy from one form to another.

Since work reflects a change in energy, work and energy have the same units: Joules.

(Quick review: Impulse is a change in Momentum, and both have the same units as well, shots)

There are many forms of energy. The ones we are interested in right now are potential energy and kinetic energy. Other forms of energy, such as sound energy or heat energy are often what we experience as friction, and end up robbing us of usable energy. So we'll try to avoid them if possible.

The energy added to an object by moving it is equal to the amount of work you need to do to move the object. Work is a change in the amount of energy.

Potential Energy

Potential energy is energy stored in the form of a mass lifted vertically from a starting height. You do some work to lift a mass some vertical height, and then you can recover that work later if you want.

We already showed how much work it takes to lift a mass vertically, and that means we've shown how much our energy increases if we raise a weight, and how much our energy will decrease if we lower a weight.

Change in potential energy = mass * gravity * (change in height)

Showing the units:

potential energy (joules) = mass(kg) * gravity (m/s^2) * height (meters)

Kinetic Energy

Kinetic energy is energy stored in the form of a moving mass. You do some work accelerating some mass horizontally, which increases the kinetic energy of the object. You can then recover the work from the object at a later time if you want (and if friction hasn't slowed it down).

We already showed how much work it takes to move a mass from zero to some velocity, and that means we've already shown how much our energy increases if we speed up a mass, or how much our energy decreases if we slow down a mass.

Change in kinetic energy = ½ * mass * (change in velocity)^2

showing the units:

kinetic energy (joules) = ½ * mass(kg) * (change in velocity(m/s))^2

Law of Conservation of Energy

Liebniz was doing work with Newton's three laws and with the law of conservation of momentum and discovered that there was another fundamental law at play with objects in motion. The law is that the energy associated with an object is conserved, although it may change from one form to another. Liebniz found that objects had an energy that was related to mass * velocity^2, which we already showed is in our formula for calculating work and for calculating the kinetic energy of an object moving horizontally.

Liebniz found that kinetic energy and potential energy are two forms of energy that can occur. For a while though, other people were observing systems that would seemingly lose energy. In 1798, Count Rumford observed heat being generated when a machine was boring a cannon barrel. Soon, people realized that mechanical energy could be transferred into heat through friction. And the conservation of energy was shown to be true if all forms of energy were taken into account.

Energy is different from momentum. Momentum says that an object in motion will remain in motion at the same speed and direction until some external force acts on it. Energy is the ability to exert a force on some object over a distance. Momentum or inertia exist without any forces.

It is somewhat non-intuitive to realize that these two concepts are fundamentally different. Both involve formulas that use mass and velocity. But we'll show later during some experiments with collisions that the formula for an objects momentum and an objects energy are independent equations and allow us to solve problems with two unknowns. For now, just take my word for it that momentum and energy are different animals.

Comparing Potential and Kinetic Energy: Superball

According to the conservation of energy law, energy in a closed system can't be created or destroyed, it can only change form. So, we have a superball, one of those really bouncy rubber balls that people play with. We'll assume the superball is super elastic. That means we'll assume that when it bounces, it doesn't convert much energy into heat or sound or other versions of friction that will rob us of a super bouncy ball. The superball weighs 0.1 kilogram. We get a step ladder and drop the superball from a height of 3 meters above a really smooth, hard floor.

What happens?

The work we do lifting the superball up the ladder is equal to m*g*h:

potential energy = 0.1 kg * 10 m/s^2 * 3 m = 3 joules

The ball will drop with constant acceleration of 10 m/s^2. How long will it take to drop 3 meters?

The formula for distance traveled with constant acceleration is:

distance = ½ * acceleration * time^2

Putting in the numbers we know:

3 meters = ½ * 10 * time^2

solving:

3 = 5 * time^2
time^2 = 3/5
time = sqrt(3/5)
time = 0.7746 seconds.

If we accelerate at 10 m/s^2 for 0.7746 seconds, how fast will we be going?

velocity = acceleration * time
velocity = 10 * 0.7746
velocity = 7.746 m/s

So if we drop any mass from a height of 3 meters, it will take .77 seconds to reach the floor, and it will be traveling at 7.7 meters per second when it hits the floor. This will be true for any mass. Why? Remember the races with Buckaroo's skateboards? Everyone ties, no matter how much they weigh. All mass falls in gravity at the same acceleration, regardless of how much mass it is.

So, what is the kinetic energy of a ball weighing 0.1 kg, traveling at 7.746 m/s?

kinetic energy = ½ * mass * veloctity^2
ke = ½ * 0.1 * 7.746^2
ke = 3 joules

How much work did it take to lift the 0.1 kg superball up 3 meters? Remember? It took us 3 joules.

It took us 3 joules to raise the ball from the floor. And once we dropped it, it had 3 joules of kinetic energy when it hit the floor again.

Using simple distance/velocity formula for an object traveling at constant acceleration, we calculated a velocity that fits our formula for kinetic energy. A 0.1 kilogram ball has 3 joules of potential energy when it is 3 meters above the floor, and it has 3 joules of kinetic energy with it has fallen 3 meters and is moving 7.746 m/s.

What happens next? Bumbles bounce.

The superball hits the floor and most of its energy is maintained in the kinetic energy of the ball, very little is absorbed by the ball in the form of heat or sound. So the ball is now heading straight up with 3 joules of energy.

When will the ball bleed off all of its kinetic energy?

The ball will keep rising until 3 joules of kinetic energy are bled off into potential energy, at which point the velocity of the ball will be zero. That will occur just below where you released the ball, 3 meters above the floor, 0.7746 seconds after it hits the floor.

The thing to keep in mind is you gave the ball 3 joules of energy when you lifted it up the step ladder. But once you released the ball, it and the floor became a closed system, and no more energy entered the ball.

From the moment you let go of the ball, the ball became a *closed* energy system.

No more energy entered the ball. It just started changing forms. From potential energy to kinetic and back to potential energy again. It will repeat this until the friction of the air and within the ball eventually bleed off the energy in the form of heat and sound and other forms.

Put in even shorter form:

At every stage while it is bouncing up and down, the superball has the same energy. It may be changing velocity, but it is also changing height above the earth, so the total energy within the ball remains constant.

This is the conservation of energy at play.

You added 3 joules of energy to the ball when you lifted it off the floor. Once you let it go, it kept those 3 extra joules of energy as it bounced up and down, back and forth, over and over again. The only thing that pulled that energy out of the ball was that friction slowly converted bits of energy into heat as it moved through the air and as it bounced against the floor. By the first bounce, it might have 2.999 joules of extra energy, the 0.001 joules being lost to air friction. After the first bounce, it might have 2.994 because the bounce absorbed 0.005 joules of energy. The bouncing of the ball, the speeding up and slowing down and changing direction, are all reflections of the ball maintaining constant energy. It is the slow decay of the bounces due to friction that eventually brings the ball to a halt.

The momentum of the ball is changing every instant as its velocity changes. But the total energy of the ball remains constant as soon as you let go. If it were a perfectly elastic ball, bouncing in a vacuum, and any other sources of energy bleeds were removed, it could bounce forever.

The real world equivalent is a satellite orbiting Earth. The satellite changes velocity every instant, causing it to continuously change its momentum. But while its velocity changes, its total energy remains constant.

There is a force between the earth and the satellite, but the force is perpendicular to the direction of travel of the satellite, therefore no work is done. And since work indicates a change in energy, and work is zero, then energy change is zero.

This is why work is defined as a force in a direction of travel. Because there is no energy change when the force doesn't occur in the direction of movement.

In physics, work is defined as a change in energy, which ends up being a force exerted over a distance. If you push on an object and it doesn't move, then you haven't changed the energy of the object, so you haven't done any work in the physics sense of the word.

Work is a change of Energy. (by comparison and for review, Impulse is a change in Momentum.)

Derived Unit: Power = The Rate at Which Work is Done = Watts

Our next derived unit is called “Power”. Power is defined as the rate at which work done, or work per second, or energy consumed or generated per second. The SI unit for power is “watts”.

Power (watts) = work done per second

Work is defined as joules.

Power (watts) = joules / second

A joule is a force of 1 newton exerted over a distance of 1 meter.

power (watts) = (newton * meter) / second

One newton is one pshooter is one “shot” per second

power (watts) = (shot/second) * meter / second

A shot is an impulse of momentum, which is 1 kg*meter/sec.

power(watts) = ([kg*meter/sec]/sec) * meter/sec

Sorting out all the units

power (watts) = kg * meter^2 / sec^3

That jumble of units is totally nonintuitive to me. I have to think of power as work per second, and think of work as some number of pshooters * the distance those pshooters move some mass.

Those familiar with electrical equipment know that electrical power consumption is usually given in watts. And it happens that electrical power in watts equals 1 volt flowing at 1 ampere of current.

power(watts) = volts * amps

Which makes for easy conversions.

Powered Deck of Cards

Remember our deck of cards exercise? A deck of cards weighs about 0.1 kilograms, so earth's gravity will exert about 1 newton of force on it. If you take a deck of cards on the floor and lift it one meter to put it on a table, you've done 1 joule of work.

So, how do we translate that to power? Well, power is joules per second, and 1 deck of cards from floor to table is 1 joule. So, if you lift 1 deck of cards from floor to table in 1 second, you are exhibiting 1 watt of power. If it takes you 10 seconds, then your power rating is 0.1 watts. If you can do it in one-tenth of a second, then you have a power rating of 10 watts.

You could play around with different amounts of mass too. You might put one hundred decks of playing cards in a box (about 10 kilograms), and then see how fast you can lift the entire box one meter. If you lift 10 kg 1 meter in 1 second, then you're power rating would be about 100 watts.

Those familiar with the English unit of power, the horsepower, might wonder how to translate the two. Turns out that 1 horsepower is about 746 watts. How did someone come up with that number? That someone was James Watt. One story is that James Watt was trying to compare his steam engines to horses that were being used to lift coal out of a mine. Watt found that a pony could lift 220 pounds of coal at a vertical velocity of 100 feet per minute, and do so for a four-hour work shift.

220 pounds = 100 kg.

100 feet per minute = 30.5 meters / minute = 0.508 meters / second

assume standard gravity of 9.8 meters/sec/sec

power = 100kg * 0.5 vertical meters / second * 9.8m/s^2 = 498 watts.

James Watt then estimated that a full grown horse had 50% more power than a pony.

490 watts * 1.5 = 746 watts

For his excellent work with horses, and some dealings with steam engines, James Watt's last name became the SI unit of power, the “watt”.

For those who are curious, an average human being can sustain a power rating of about 0.1 horsepower. Athletes can sustain about 0.3 horsepower.

Power is a Mass Moving at Constant Vertical Velocity

Momentum is a mass moving at a velocity. If you've got fixed momentum, then you've got fixed velocity (this assumes constant mass, of course, but hockey pucks don't change mass too often.) And if you have fixed velocity, your mass must be moving somewhere in deep space or horizontally over the surface of a planet so that gravity can't speed it up or slow it down.

Power is the rate of work being done. If you have a fixed power rating, then you can do a fixed amount of work every second. Fixed power has a bunch of different ways of manifesting itself, but if you want to relate it in a way similar to fixed momentum, then the easiest thing to think of is taking a fixed mass in standard gravity and moving it at a fixed vertical speed.

Moving a deck of cards one vertical meter in standard gravity is about 1 joule of work. Lifting that deck of cards one meter every second is about 1 watt of power.

One horsepower can lift 746 decks of cards (74 kg) one vertical meter every second.

If you're stuck near the surface of the earth, it may be useful to think of momentum as a mass moving at fixed horizontal velocity and think of power as a mass moving at a fixed vertical velocity.

But Kilowatt-hour is Energy

And just to confuse people, there is the watt-second or watt-hour or kilowatt hour. What the heck is that all about? If you consume 1 kilowatt of power for 1 hour, you have consumed one kilowatt hour, and your power company usually charges you some amount of money per kilowatt hour. But even though it comes from your power company, a kilowatt-hour isn't a measure of power. It's a measure of energy.

A watt is a measure of energy consumed per second, where energy is measured in joules.

1 watt of power = joules / second

If you consume energy at some number of joules per second, and you do so for 3600 seconds, what are you left with?

some number of (joules/second) * 3600 seconds = ???

The seconds cancel out, and you're left with joules, but multiplied by 3600, since there are 3600 seconds in an hour, and the power company just charged you for some number of kilowatt hours. So, 1 kilowatt hour is equal to 3.6 megajoules of work or energy done. (you did multiply by 1000 to adjust for the “kilo” in front of “kilowatt”, right?)

1 kilowatt-hour = 3.6 megajoules of energy or work

So turn those lights off when you're not in the room.

Review: Impulse,Momentum,Force,Work,Energy,Power

Time for a review of everything thus far, and maybe make some round about connections.

Momentum is the mass of some object multiplied by its velocity. Impulse is a change in momentum. We also call impulse a “shot”. If you fire a constant rate of shots per second, you exert a force on something. We also call “shots per second” a “pshooter”. Momentum is always conserved.

When you exert a constant force on a mass, you get constant acceleration. If you exert the same force on a different mass, you get a different acceleration. If the force you exert is a function of the mass of the thing you're pushing on, then you get constant acceleration regardless of the mass of object involved. This is how Buckaroo's skateboard works. Gravity works this way too, giving all objects the same rate of acceleration, regardless of their mass.

We then defined work to be a force exerted on a mass over some distance, which is measured in joules. We found that the work required to lift an object was equal to mass * gravity * height. We also found that the work needed to accelerate some object from rest to some velocity is equal to ½ * mass * velocity squared.

We then defined energy to be the ability to do work at a later time, the ability to exert a force over some distance. We then defined work to be an indication of a change in energy. Without some energy, you can do no work. And work indicates the amount of energy you've used or produced.

The conservation of energy says that a closed system cannot create or destroy energy, only change its form. We then showed that we could do work to a superball to lift it, then release it and watch it bounce over and over again, changing potential energy into kinetic energy and back to potential energy, but seeing that during the entire time it is bouncing, it is also keeping its total energy at a constant.

We then defined power as the rate at which work is done or the rate at which energy is produced or consumed. The SI unit for power is watts. 1 watt is 1 joule per second. 1 horsepower is 746 watts. In standard gravity, constant power is a function of moving some constant mass at some constant vertical velocity. To lift a deck of cards at a vertical velocity of 1 meter per second would require about 1 watt of power.

Lastly, a kilowatt-hour is a measure of energy (3.6 megajoules of energy), not a measure of power.

Review, Part 2: For Us Earth Bound Types

For those of us dealing in the day to day issues of living on the surface of a standard Class M planet, we can think of the concepts we've covered so far in somewhat more intuitive terms.

Momentum is a mass moving horizontally with no friction. (a puck on our air hockey table)

Impulse is a change in momentum. (a “shot” applied to puck on air hockey table)

Force causes a change in momentum (some number of shots per second, a “pshooter”)

Work is a force applied over a distance (horizontally: buckaroo's skateboard, vertically: lifting a mass)

The energy of an object is changed by the work you do on it (don't forget, you can give it kinetic or potential energy)

Power is energy per second. Constant power translates into lifting a mass at a constant vertical velocity.

Complicated Collisions: Momentum and Energy

When we discussed conservation of momentum, we presented a number of examples of collisions that demonstrated basic momentum conservation. One of the simplest is two billiard balls colliding on a billiard table. Another example was two hockey pucks colliding on our air hockey table. Aanother example was a “Newton's Cradle”, a toy with a number of steel sphere's hanging in a line.

All of those examples of collisions were demonstrations to show how conservation of momentum worked, and we calculated the results of those collisions using only the law of conservation of momentum. Momentum = mass * velocity.

But since then, we've shown that there is a law of conservation of energy, and kinetic energy of a moving object is ½ * mass * velocity^2. So, how did we manage to do our collision examples and have them work out correctly using only momentum calculations without doing calculations to make sure energy was conserved too?

We cheated.

We rigged the numbers.

All of our collision examples involving momentum involved object of equal mass. Billiard balls all weigh the same. Hockey pucks all weigh the same. The spheres in Newton's cradle all weigh the same.

What difference does that make?

Say you've got our cue ball on the billiard table and you're trying to sink the 8 ball. The velocity of the cue ball before the collision equals the velocity of the 8 ball after the collision. And we can solve this using only conservation of momentum numbers. What happens to the energy formula? If the mass of the cue ball and 8 ball are the same, then the conservation of energy formula says the velocity before and the velocity after will be the same as well.

Because conservation of energy and conservation of momentum give the same velocities answers when the masses of the objects are all the same, we can calculate the answers using only the conservation of momentum.

But what if the masses are different?

Say we take our air hockey table. We've got a puck that weighs about 150 grams. And we tape two pucks together for a double puck weighing 300 grams. We start off the cue puck with a velocity of 1 meter per second and aim it squarely at the center of the double puck. What happens?

The first question is “Can we assume the puck will stop after it hits the double puck?” We've been assuming that in our previous examples, but all of our previous examples had the collider and collidee have the same mass. In this new example, the masses are different, so we can't really assume the puck will completely stop.

Well, we could assume that, but lets see what happens.

Assuming Puck Stops After Collision

Start with conservation of momentum. Before the collision, only the single puck is moving. After the collision, assume only the double puck is moving. So the momentum of the single puck before the collision must equal the momentum of the double puck after the collision.

0.150 kg * 1 m/s = 0.300 kg * ?v?

solve for v

v = 0.150 * 1 / 0.300 = 0.5 meters per second

Intuitively, that sounds right. The double puck is twice as massive as the single puck, so the double puck moves at half the speed of the single puck.

But now we know that we have a law of conservation of energy. Energy must always be conserved. Was it? We assume the cue puck was the only thing moving before the collision and the double puck was the only thing moving after the collision. So, compare energy of single puck with energy of double puck and they should match.

kinetic energy = ½ * mass (kg) * velocity squared (m/s)
ke single puck = 0.5 * 0.150 kg * 1^2 m/s = 0.075 joules
ke double puck = 0.5 * 0.300 kg * 0.5^2 m/s = 0.0375

Whoops. Somehow we lost energy in the system. And we know conservation of energy says we can't just lose energy, and we're pretty sure that we didn't lose energy through friction or heat or something. So, maybe our assumption that the single puck stops moving is wrong.

What do we do?

Two Equations, Two Unknowns

We have two laws that must always be observed: conservation of momentum and conservation of energy. These two laws give us two equations. The two unknowns are the velocity of the single and double puck after the collision. We don't know how fast they will be moving. We tried to assume the single puck would stop moving after the collision, and use conservation of momentum to figure out the velocity of the double puck, but when we plugged that velocity into the energy equations, we found that we lost a lot of energy somewhere.

But if we look at the conservation of momentum as one equation, and the conservation of energy as another equation, and we only have two unknowns (velocity of two pucks after collision), then we can use algebra to solve the problem. Two equations and two unknowns.

The momentum and energy of the system before the collision is given by the velocity of the single puck because the double puck isn't moving.

momentum before collision = m*v = 0.15 kg * 1 ms/ = 0.15 shots
kinetic energy before collision = 1/2*m*v^2 = 0.5 * 0.15 kg * 1^2 m/s = 0.075 joules

Momentum and kinetic energy must be conserved. So, what are the momentum and kinetic energy of the system after the collision? Assume that after the collision, we use v1 as velocity of single puck and v2 as velocity of double puck.

momentum = sum of (mass * velocity) of individual items
kinetic energy = sum of (½ * mass * velocity^2) of individual items

Plugging in the numbers that we do know.

momentum = 0.15 shots = [0.15 * v1] + [0.30 * v2]
k.e. 0.075 joules = [½ * 0.15 * v1^2] + [½ * 0.30 * v2^2]
ke => 0.075 = 0.075*v1^2 + 0.15*v2^2

We can take the momentum equation and solve it for v1.

momentum 0.15 = [0.15 * v1] + [0.30 * v2]
momentum 0.15 - [0.30 * v2] = [0.15 * v1]
v1 = (0.15 - [0.30 * v2])/0.15
v1 = 1 - 2*v2

Now we can substitute this value of v1 into the kinetic energy equation

0.075 joules = 0.075 * (1 – 2*v2)^2 + 0.15 * v2^2

we can substitute (a-b)^2 = a^2 - 2ab + b^2

0.075 joules = 0.075 * (1 - 4*v2 + 4*v^2) + 0.15 * v2
0.075 joules = 0.075 - 0.30*v2 - 0.30*v^2 + 0.15 * v2
0.45*v2^2 – 0.3*v2 = 0

The quadratic formula says that given A*x^2 + B*x + C = 0, then x = [-B +/- sqrt(B^2 – 4*A*C)]/2*A

v2= [0.30 +/- sqrt(0.3^2 – 4*0.45*0)] / 2 * 0.45
v2 = [0.3 +/- sqrt (0.3^2)] / 0.9
v2 = [0.3 +/- 0.3] / 0.9
v2 = 0 or .66666667

The velocity of the double puck is zero or .6667 meters per second. It was zero before the collision, so it must be 0.666667 m/s after.

Taking that result and plugging it into the momentum formula:

momentum = 0.15 shots = [0.15 * v1] + [0.30 * v2]
=> 0.15 = 0.15*v1 + 0.30 *0.666667
=> 0.15 * v1 = 0.15 – 0.20
=> 0.15 * v1 = -0.05
v1 = -0.05 / 0.15
v1 = -0.33333

The velocity of the double puck is 0.66667 meters/second. The velocity of the single puck is -0.3333. A negative number means the puck recoiled and ended up traveling in the opposite direction.

So, the first thing we can do with numbers is confirm momentum and energy were conserved:

momentum before collision = m*v = 0.15 kg * 1 ms/ = 0.15 shots
momentum after collision = (0.15 * -0.3333) + (0.30 * 0.66666)
momentum after = -0.05 + 0.2 = 0.15
energy before collision = 1/2*m*v^2 = 0.5 * 0.15 kg * 1^2 m/s = 0.075 joules
energy after collision = ½ * 0.15 * (-0.33333)^2 + ½ * 0.30 * 0.666666^2
energy after = 0.075 * 0.11111 + 0.15 * 0.444444
energy after = 0.00833333 + 0.06666667
energy after = 0.075 joules

Momentum and energy appear to have been conserved, so the numbers seem to be accurate.

Starting with an air hockey puck weighing 150 grams, and shooting it with a velocity of 1 meter per second at a double-puck weighing 300 grams, in a head on collision, results in the double puck moving forward at 0.6667 meters per second and the single puck moving *backwards* at 0.3333 meters per second.

Somewhat Non Intuitive Results

If you're used to billiards, this answer (double puck=0.6666 m/s, single puck=-0.3333 m/s) may seem un-intuitive.

The thing is that there are always two basic laws at play in collisions, the conservation of momentum and the conservation of energy. These two laws have two independent equations that must be satisfied. And this yields one, and only one, possible solution.

Think about it, if conservation of momentum were the only limiting requirement, then you'd have the velocities of two objects after a collision that are unknown and only one equation to solve for them. That would mean the exact same collision could have an infinite number of valid after-collision velocities for the two masses. As long as the total momentum after the collision added up to equal the total momentum of the system before the collision, you could have any combination of velocities that could work together to balance the equation and conserve momentum. If you start with a momentum of 1 kg*m/s, and have a collision, then if one object moved off at a million meters per second, that would be fine as far as momentum is concerned as long as the second object moved at minus one million and one meters per second. The total momentum would still be 1 kg*m/s.

Instead, the conservation of energy law is another set of requirements that results in only one possible outcome for a collision. There is only one combination of velocities for the two objects that conserves both momentum and energy after the collision. And that is how the objects behave.

Velocity of two masses after Elactic Collision

Given two objects colliding elastically and given the second object is not moving, we can simplify the formula for the final velocities. A nice formula from this website:

http://teacher.nsrl.rochester.edu/PhyInq/Lectures/Collisions/Collisions.html

Here are the equations:

v1final = v1initial * (m1 – m2) / (m1+m2)
v2final = v1initial * (2 * m1) / (m1+m2)

solving for our puck/doublepuck example:

v1final = 1 * (0.15-0.3)/(0.15+0.3) = (-0.15)/(0.45) = -0.3333 m/s
v2 final = 1 * (2*0.15) / (0.15+0.3) = (0.3)/(0.45) = 0.6666667 m/s

The single puck bounces back and moves backwards at-0.333 m/s.

The double puck moves forward at 0.66667 m/s.

So, the shortcut formulas appear to work.

Elastic Versus Inelastic Collisions

We've mentioned “elastic” collisions previously. What exactly does that mean?

Elastic means that the kinetic energy in the objects about to collide is conserved and is equal to (or nearly equal to) the kinetic energy of the objects after the collision.

Bouncing a superball on a hard floor is an elastic collision. The steel spheres clacking together in a Newton's Cradle toy are elastic collision. The energy is conserved.

So what is an inelastic collision?

An inelastic collision is a collisions where kinetic energy is not conserved. Instead it is absorbed and converted into heat energy and other forms. When two cars crash and the frame of the car is deformed and crumpled zones are crumpled, those deformations took energy.

Car crashes are inelastic collisions. Any sort of collision where the two objects stick together after the collision is an inelastic collision. A common physics question involving inelastic collisions involves shooting a bullet into a block suspended on a string. The block absorbs the bullet, and then the block and bullet swing on the string.

Note that even if a collision is inelastic and kinetic energy is not conserved, conservation of momentum must still be followed.

Therefore, the approach to solving a problem involving an inelastic collision is to use conservation of momentum as a starting point for the solution.

Inertial Frames

In Newtonian physics, an inertial frame is any frame of observation in which Newton's three laws of motion are observed to be valid.

Momentum is conserved, force = mass * acceleration, for every action there is an equal and opposite reaction, and energy is conserved.

Any fixed point on the surface of the earth approximates an inertial frame for many problems, but not all problems. If you're calculating billiard ball collisions on the surface of the Earth, you may consider the billiard table on the surface of the Earth to be an inertial frame.

If you're calculating the position of Focault's pendulum, i.e. a 28 kilogram bob, suspended from 67 meters of wire, hanging from the ceiling of the Pantheon in Paris, then you would find that your pendulum not only swings back and forth, but it appears to “rotate”. It appears to rotate because the Earth is rotating underneath it while the pendulum is swinging in a fixed plane. At which point, the surface of the Earth is not an inertial frame.

But for many physics problems, you may assume that any fixed point on the surface of the Earth is an inertial frame.

If a fixed point on the surface of the Earth is an inertial frame, what else is?

It turns out that any point on the Earth's surface moving at a fixed velocity and remaining at a fixed altitude is also an inertial frame. It has to be a fixed altitude because if the point moves up and down, observers on that frame will observe objects that appear to move on their own, without any apparent force applied to them. And it has to be moving at constant velocity because if it is accelerating, then it will appear that a force is acting on any objects in the frame.

What's the purpose of inertial frames?

They're an attempt to make your physics problems easier. The fact that you can calculate the trajectory of an artillery shell fired from a point on the Earth's surface using the Earth's surface as an inertial frame makes your calculations a whole lot easier. You don't have to deal with the Earth's rotation, its orbit around the sun, and the sun's orbit through the galactic disc.

But if you place an artillery cannon on a train, have the train moving at a fixed velocity, and fire the cannon, the trajectory of the shell relative to the train will be exactly the same as the trajectory of the shell relative to the fixed point on the earth when the cannon was at a stationary point on the Earth's surface.

Rotational Momentum

tbd

Rotational Kinetic Energy

tbd

Resources

The following pages are for resources that readers can use if they're so inclined.

Shots Per Second Calculator

```
#!/usr/local/bin/perl
use warnings; use strict;

# this script is used to simulate an impulse being applied to a mass at some fixed rate
# It prints out the simulation time, the velocity of the object being moved, and the total distance
# the object has traveled. If you divide a second up into a sufficiently large number of mini-shots,
# then the motion of the object approaches an object with a constant force on it.

my $momentum_per_second=2;              # change this to whatever impulse per second you want
my $shots_per_second=10;                # this is the "resolution" of the simulation
my $total_seconds_to_simulate=10;       # how long do you want to simulate?

my $momentum_per_shot = $momentum_per_second/$shots_per_second;
my $simulation_time_resolution = 1/$shots_per_second;

my $velocity=0;
my @instants=( [0,0] );

for(my $time=$simulation_time_resolution; $time<=$total_seconds_to_simulate;
$time+=$simulation_time_resolution) {
        $velocity += $momentum_per_shot;
        push(@instants,[$time,$velocity]);
}

my @distances;
my $distance=0;
foreach my $entry (@instants) {
        my ($time,$velocity)=@$entry;
        $distance+=($velocity*$simulation_time_resolution);
        push(@$entry,$distance);
}

my $lasttime=0;

foreach my $entry (@instants) {
```

```
        my ($time, $velocity, $distance) = @$entry;
        if(int($time)>$lasttime) {
                printf "time=%4.2f, velocity=%6.1f, distance=%6.1f\n",
                $time,$velocity,$distance;
                $lasttime=int($time);
        }
}
```

Shots per second output example

Here's the output of the shots per second calculator script run with the values shown in the code example above:

```
time=1.10, velocity=   2.2, distance=   1.3
time=2.00, velocity=   4.0, distance=   4.2
time=3.00, velocity=   6.0, distance=   9.3
time=4.00, velocity=   8.0, distance=  16.4
time=5.10, velocity=  10.2, distance=  26.5
time=6.10, velocity=  12.2, distance=  37.8
time=7.10, velocity=  14.2, distance=  51.1
time=8.10, velocity=  16.2, distanceEnter value=  66.4
time=9.10, velocity=  18.2, distance=  83.7
```

You can see that we can calculate that having a mass and applying some fixed number of shots per second to it will move that mass at a fixed acceleration. And you can see the distance the object traveled is approaching the formula:

distance = ½ * acceleration * time^2

English Conversion Units

When you do physics, do your calculations in SI units. If the problem gives you numbers that are in English Units, convert them to metric first then go from there.

What follows are conversion tables for English units

Length.

1 foot = 0.3408 meters

1 digit	¾ inch	0.01905 meters
1 finger	7/8 inch	0.022225 meters
1 hand	4 inches	0.1016 meters
12 inches	1 foot	0.3408 meters
1 nail	3 digits	0.05715 meters
1 palm	3 inches	0.0762 meters
1 span	9 inches	0.2286 meters
1 foot	12 inches	0.3408 meters
1 cubit	forearm, 18 inches	0.4572 meters
1 yard	3 feet	0.9144 meters
1 ell	45 inches	1.143 meters
1 fathom	6 feet	1.8288 meters
1 rod	16.5 feet	5.0292 meters
1 chain	4 rods, 66 feet	20.1168 meters
1 furlong	660 feet	201.168 meters
1 mile	8 furlongs 5280 feet	1609.344 meters
1 league	3 miles	4828.032 meters

Area:

1 perch	1 square rod 16.5 feet*16.5 feet 272.25 ft^2	82.9818 square meters
1 acre	43560 square feet	13227.088 square meters
1 rood	quarter acre	3319.272 square meters
1 carucate	120 rood	398312 square meters
1 bovate	15 roods	49789.08 square meters
1 virgate	30 roods	99578.16 square meters

volume:

1 gallon = 3.7853312 liters

1 cubic meter = 1000 liters

1 mouthful	½ ounce	0.01478645 liters
1 jigger	1 ounce	0.0295729 liters
1 jack	2 ounce	0.0591458 liters
1 jackpot	2 ounce	0.0591458 liters
1 gill	4 ounce	0.1182916 liters
1 cup	8 ounces	0.2365832 liters
1 pint	2 cups, 16 ounces	0.4731664 liters
1 quart	2 pints, 32 ounces	0.9463328 liters
1 gallon	4 quarts 64 ounces	3.7853312 liters
1 peck	2 gallons	7.5706624 liters
1 kenning	4 gallons	15.1413248 liters
1 bushel	8 gallons	30.2826496 liters
1 cask, 1 strike, 1 coomb	2 bushels 16 gallons	60.5652992 liters
1 barrel	2 casks, 32 gallons	242.2611968 liters
1 hogshead	64 gallons	484.5223936 liters
1 tun	256 gallons 2,048 pounds (about 1 ton)	1938.0895744 liters

Weight (Avoirdupois system)

1 avoirdupois pound = 0.4535924 kilograms

1 grain (gr)	64.79891 mg 1/7000 pound	0.000064799 kilogram
1 dram	1/16th of an ounce	0.001771845 kilograms
1 ounce (oz)	16 drams 437.5 grains 1/16th of a pound ~28 grams	0.028349525 kilograms
1 pound (lb) (lb is short for "libre")	16 ounces 7000 grains 454 grams	0.4535924 kilograms
1 hundred-weight (cwt)	112 pounds (long) 110 pounds(short)	50.8023488 kilograms (long) 49.895164 kilograms (short)
1 ton	20 cwt	907.1848 kilograms (short) 1016.046976 kilograms (long)
1 nail	7 pounds	3.1751468 kilograms
1 stone	14 pounds	6.3502936 kilograms
1 tod	2 stones	12.7005872 kilograms

Weight (troy system)

1 grain	64.79891 mg	0.000064799 kilogram
1 pennyweight	24 grains 1.56 grams	0.001555176 kilograms
1 troy ounce	20 pennyweight	0.03110352 kilograms
1 troy pound	12 troy ounces	0.37324224 kilograms
1 mark	8 troy ounces	0.24882816 kilograms
1 scruple	20 grains	0.00129598 kilograms
1 dram	2 scruples	0.00259196 kilograms
1 apothecary ounce	8 apothecary drams 480 grains	0.03110352 kilograms
1 apothecary pound	5760 grains	0.37324224 kilograms

Oddball conversions

These are just some assorted, but somewhat common, units you might run into.

1 foot pound

As a unit of torque, 1 foot pound of torque is the torque created by one pound of force exerted 1 foot from the axis of rotation. 1 foot pound = 1.35581794 newton-meters

As a unit of energy, 1 foot pound is the work done by a force of one pound exerted over a distance of one foot. 1 foot pound of energy = 1.35581794 joules

1 foot pound = 192 inch ounce

1 horsepower

A unit of power. 1 horsepower = 330,000 ft-lb/min = 550 ft-lb/sec = 745.6999 watts

1 watt = 1 newton-meter/second

1 newton = 1 kg-m/s per second

British Thermal Units

A BTU is a measure of energy. 1 BTU approximately equals:

1060 joules

2.93e-4 kilowatt hours

252 “little” calories

0.25 “large” calories (food calories)

1 atmosphere

a unit of pressure (force per square distance)

1 atmosphere = 101,325 pascals

approximated as 1 atmosphere = 1 bar, where 1 bar = 100,000 pascals

1 pascal is 1 newton of force per square meter

1 atmosphere can also be converted to:

1.01325 bar

1013.25 hectopascal

1013.25 milllibars (mbar or mb)

760 torr

760 millimeters of mercury

29.9213 inches of mercury

1.033227 kilogram force / centimeter square

1033.227 centimeters of water

406.782 inches of water

14,69595 pounds-force per square inch (psi)

2116.2166 pounds-force per square foot